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When shot-noise-limited photodetectors disobey Poisson statistics

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Photodetectors with internal gain are of great interest for imaging applications, since internal gain reduces the effective noise of readout electronics. High-gain photodetectors have been demonstrated, but only individually rather than as a full array in a camera. Consequently, there has been little investigation of the interaction between camera complementary metal oxide semiconductor (CMOS) electronics and the slow response time that high-gain photodetectors often exhibit. Here we show that this interaction filters shot noise and causes noise statistics to differ from the common Poisson distribution. As an example, we investigate a 320 × 256 array of InGaAs/InP high-gain phototransistors bonded to a CMOS readout chip. We demonstrate the filtering effects and discuss their consequences, including new (to the best of our knowledge) methods for extracting gain and increasing dynamic range.

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There are many applications that require very sensitive imaging at high frame rates. One example is extreme adaptive optics for the imaging of exoplanets, which requires operating at extremely low light levels and frame rates exceeding 1000 frames per second [1]; many other instances inside and outside of astronomy are similarly "light-starved." Imaging requires a camera array rather than a single-element photodetector, which in turn requires a readout integrated circuit (ROIC) to capture and transmit the output of each pixel for digitization. The electronic noise generated by the ROIC's operation prevents shot-noise-limited performance in low-light conditions. An increasingly attractive solution is to use pixels with internal gain, which amplify the photo-response above this ROIC read noise. The limiting noise is then the shot noise in the pixel's dark current.

This shot noise usually follows the square root of the number of carriers. However, Poisson statistics apply only when the particles' arrival times are independent. The output of a photodetector is uncorrelated only for timescales much longer than its response time τ. If the integration time T of a camera readout is significantly longer than τ, consecutive frames are independent. We refer to this regime as "ROIC limited," since the ROIC integration time determines the system bandwidth. However, in the "detector-limited" regime where $T < 2\tau$, adjacent frames are correlated, and Poisson statistics do not apply. In effect, the system noise is filtered by the response time of the photodetector pixel.

Up until now, operation in the detector-limited regime has been rare. Benchtop amplifiers and spectrum analyzers operate on timescales much longer than the τ of single-element photodetectors. Camera readout integration times can be as short as microseconds, but until now all camera photodetectors have responded even faster. However, many new types of photodetectors with high internal gain have much slower response times than older low-gain designs. When used as camera pixels, these photodetectors will exhibit filtering effects and violate Poisson statistics. Thus, understanding the filtering is critical to properly predicting the sensitivity of cameras made from shot-noise-limited photodetectors with high internal gain.

This filtering effect is especially important since the internal gain G itself is commonly characterized using shot noise [2–5], as it is impossible to measure it directly. Comparing the number of output electrons to the number of incident photons gives only the external gain G_{ex}, since not every photon contributes to the output. The two gains are related by $G_{ex} = G * QE * FF$, where the quantum efficiency QE determines the fraction of incident photons that generate carriers, and the fill factor FF is the fraction of the photodetector area where a generated carrier will contribute to the output current. There is a large body of literature on extracting QE and FF, but these approaches assume discrete components where the signal can be measured at different locations within the measurement circuit. They cannot be applied to integrated arrays. And high-gain photodetectors tend to have low FF, so a method to directly extract G is even more important. We show that noise statistics can be used to find G even for a fully integrated camera array exhibiting noise filtering effects, though the process requires understanding shot-noise statistics in the detector-limited regime.

We first present the theory of shot noise in the two noise regimes and confirm it via measurements. We have developed and fabricated an array of 320 × 256 high-gain InGaAs/InP

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phototransistors, which we have hybridized with a ROIC (FLIR ISC9705) using a conventional indium bump-bonding method. We present a robust methodology to extract the internal gain of pixels on our bonded array by taking advantage of the filtered shot noise. Finally, we show in both theory and experiment that in the detector-limited regime, the signal-to-noise ratio (SNR) at constant illumination is independent of integration time.

The most general form for expressing the shot noise current due to a current I_{in} is

$$i_{\mathrm{in}} = \sqrt{2q I_{\mathrm{in}} \mathrm{BW}}, \quad (1)$$

where q is charge per charge carrier, and BW is system bandwidth. Both current I_{in} and noise current i_{in} are internal quantities (currents occurring within the photodetector) and cannot be measured directly. Only the external quantities I_{ex} and i_{ex}, which are the outputs of the photodetector after the internal values have been amplified by the internal gain G, are physically observable. The conversions are given by

$$I_{\mathrm{ex}} = G I_{\mathrm{in}} \text{ and } i_{\mathrm{ex}} = G\sqrt{F} i_{\mathrm{in}}. \quad (2)$$

The noise conversion includes the excess noise factor F, which accounts for fluctuation of the gain mechanism itself [6]. In terms of measurable outputs, then, the external shot noise current is

$$i_{\mathrm{ex}} = \sqrt{2q(FG) I_{\mathrm{ex}} \mathrm{BW}}. \quad (3)$$

In a system consisting of a photodetector and a readout electronic circuit, BW is limited by either photodetector bandwidth or ROIC bandwidth. The photodetector response can be approximated as a single pole system with a time constant τ (the time required by the photodetector to respond at the $1/e$ level). The signal bandwidth of the photodetector is then $1/(2\pi\tau)$; but here we want the noise bandwidth, which is $1/(4\tau)$ [7]. The ROIC integrates the pixel's output signal on a capacitor, which is equivalent to applying a low-pass filter and results in $\mathrm{BW}_R = 1/(2T)$ for integration time T [8]. Thus, the minimum of $1/(4\tau)$ and $1/(2T)$ determines the total system noise bandwidth. We designate camera systems as operating in the ROIC-limited regime when $2\tau < T$ and in the detector-limited regime when $T < 2\tau$. ROICs integrate on scales from microseconds to seconds, so camera systems can switch between regimes.

In the ROIC-limited regime, system bandwidth is $\mathrm{BW}_R = 1/(2T)$. Using $I = qN/T$ to describe current in terms of number of charges N per integration time T, Eqs. (1) and (3) become

$$\sigma_{\mathrm{in,ROIC}} = \sqrt{N_{\mathrm{in}}} \quad \text{and} \quad \sigma_{\mathrm{ex,ROIC}} = \sqrt{\mathrm{FGN}_{\mathrm{ex}}}. \quad (4)$$

The two forms of Eq. (4) are the traditional statement that shot noise σ follows the square root of particle number, except corrected for gain. In the detector-limited regime, pixel speed rather than integration time limits system bandwidth: $\mathrm{BW} = 1/(4\tau)$. Eqs. (1) and (3) instead become

$$\sigma_{\mathrm{in,det}} = \sqrt{\frac{T}{2\tau} N_{\mathrm{in}}} \quad \text{and} \quad \sigma_{\mathrm{ex,det}} = \sqrt{FG \frac{T}{2\tau} N_{\mathrm{ex}}}. \quad (5)$$

A detector-limited system has $T < 2\tau$ by definition, so this noise is less than the shot noise. The photodetector filters out high-frequency components, reducing the final output noise level.

To observe this noise reduction, we operated our camera such that the pixel shown in Fig. 1 was in the detector-limited regime ($\tau = 730$ µs at 240 K operating temperature, 4590 Hz frame rate, $T \leq 202$ µs). We took thousands of consecutive dark frames to observe the noise, repeating this process for multiple integration times but with all other settings (including frame rate) held constant. In each case, the dark values formed a Gaussian distribution, as expected, but they were narrower than predicted by the usual Poisson statistics and instead more closely followed Eq. (5). The noise is slightly higher than the filtered prediction due to the presence of flicker noise as well as ROIC noise at high frequencies (where the slow response time prevents the internal gain from effectively suppressing it).

Without knowing the external efficiency, it is difficult to measure the internal gain of a photodetector directly. Noise analysis provides an alternative to direct probing for understanding the internal amplification mechanism of these photodetectors and extracting the gain. This process does not involve incident light so the external efficiency is irrelevant. Here we investigate a high-gain photodetector's noise spectrum as part of a camera system rather than standalone.

The statistical noise spectra of various photodetectors previously have been studied via direct probing, using an amplifier and spectrum analyzer [2–5]. Nearly all photodetectors are in the ROIC-limited regime when measured with benchtop instruments, so Poisson statistics apply. The white-noise plateau in the noise frequency spectrum determines i_{ex}, and Eq. (4) finds the gain. However, the spectrum is dominated by $1/f$ noise at low frequencies and truncated by the instrument bandwidth at high frequencies, which can make locating the plateau difficult.

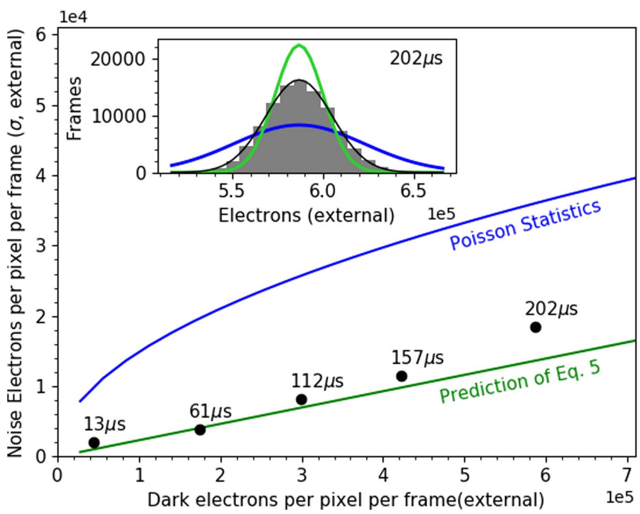

Fig. 1. Dark current electrons versus noise electrons per frame in one pixel, taken at 4590 Hz. For each integration time, a Gaussian was fit to a histogram compiled from thousands of consecutive frames to find the noise (black dots). As an example, the 202 µs measurement's histogram and fit (black line) are shown in the inset. In both plot and histogram, the measured noises are consistently less than the Poisson predictions of Eq. (4) (blue) and more closely match the detector-limited predictions of Eq. (5) (green). This is because all integration times shown lie in the detector-limited regime (the pixel's response time is 730 µs at 240 K). Both predictions used $G = 1105$ (measured as explained in section 3) and $F = 2$ (assumed).

A slow photodetector requires long integration times to remain in the ROIC-limited regime, but the high gain saturates instruments, and $1/f$ noise drowns out the plateau. The other option is to operate in the detector-limited regime, where Eq. (4) no longer applies. The relationship between current and noise becomes frequency dependent due to the pixel's filtering effect, so the concept of "white-noise level" is also inapplicable. Noise is now better described by the shape of its frequency spectrum, which can depend on the photodetector's gain mechanism.

We show that the spectrum shape can be used to find the internal gain of our camera system in the detector-limited regime. Our devices are phototransistors, which due to their capacitance require time to charge and discharge. We have recently proposed a model treating the phototransistor frequency response as a first-order filter with a 3 dB frequency $f_0 = 1/(2\pi\tau)$ for the pixel response time τ [9]. By applying a low-pass filter to Eq. (3), we find that the external noise spectrum is given by Ref. [6]:

$$i_{\text{ex}}(f) = \sqrt{\frac{2q(\text{FG})I_{\text{ex}}}{1+(f/f_0)^2}} + b. \quad \textbf{(6)}$$

Here b is the noise floor created by the ROIC readout noise, an external current. The instrument noise is not visible in ROIC-limited noise spectra because the photodetector noise dominates at all frequencies below the instrument bandwidth. However, in our detector-limited case, it appears as a high-frequency plateau where the photodetector noise is filtered out [visible in Fig. 2(b)].

All parameters in Eq. (6) besides G and b are determined directly. The excess noise factor F is two for phototransistors [6]. Response frequency f_0 is found by illuminating the pixel with a calibrated spatially uniform square-wave light pulse. The pulse is synchronized with the camera to last exactly 256 frames, allowing consecutive pulses to be averaged. This permits the use of low-intensity pulses to ensure that optical biasing does not affect pixel speed. Rising and falling exponential functions are fit to the response to yield τ and thus f_0. See Fig. 2a for an example performed at 220 K (the low temperature slows the response for better visualization). This procedure also determines external gain by observing the output change due to a known light input, but without the fill factor and quantum efficiency, the internal gain remains unknown.

The camera is then used as a sampling digital spectrum analyzer by collecting 100,000 consecutive frames at the ROIC's highest practical frame rate of 4590 Hz. The data are broken into 10 segments, a Fourier transform performed on each, and the results averaged to obtain a clean frequency spectrum. Equation (6) is then fit with gain G and ROIC noise floor b as the only free parameters. Figure 2 shows an example using a nonlinear least-squares fit, yielding a gain of 751 electrons per absorbed photon.

As an additional advantage to this method, the τ dependence in Eq. (6) pinpoints the location of the photodetector's white noise plateau even if $1/f$ noise is large. To reduce interference from $1/f$ noise, only frequencies above f_0 are used in the fit.

We have shown that the filtering effect of a photodetector's response time can be exploited to extract its gain. We now study the impact of this filtering on a camera system's ability to detect light. A system's sensitivity to a given input signal can be described by its SNR, defined as the number of output signal carriers divided by the number of output noise carriers. Say a photodetector with internal (pre-amplification) dark current I_D is illuminated to create an internal signal current I_S. From Eqs. (2) and (3), the external signal current is GI_S, and the noise in the external total current is $i_{\text{ex}} = \sqrt{2q(\text{FG}^2)(I_S + I_D)\text{BW}}$. The ROIC adds RN electrons of read noise to each frame on top of this amplified noise. Adding noise sources in quadrature and converting current to number of carriers, the SNR of a frame of integration time T is

$$\text{SNR} = \frac{N_{\text{ex,signal}}}{\sqrt{\sigma_{\text{ex}}^2 + \text{RN}^2}} = \frac{I_s}{\sqrt{2qF(I_S + I_D)BW + (\frac{q}{T}\frac{\text{RN}}{G})^2}}. \quad \textbf{(7)}$$

In the case of a system operating in the ROIC-limited regime where $\text{BW} = 1/(2T)$, SNR becomes

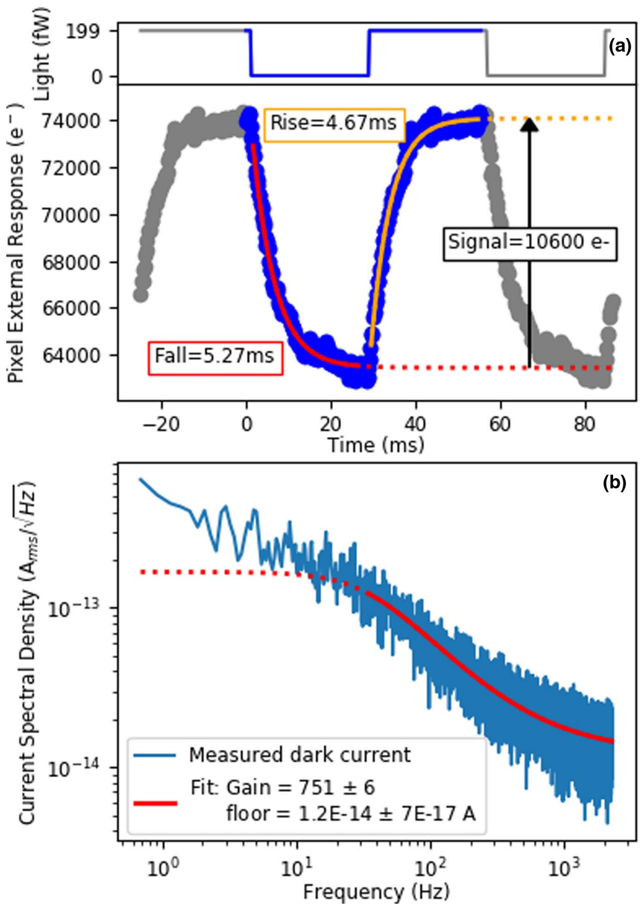

Fig. 2. Top: response of the pixel in Fig. 1 to a square-wave pulse of 200 fW light, spread uniformly across the pixel surface. Frames taken at 220 K to reduce pixel speed to better show the response, with 4590 Hz frame rate and 202 µs integration. The pulse and frame rates were synchronized to allow averaging; shown is 50 averaged responses. The known light level and measured signal yielded an external gain of 34. Dark current was 63,400 electrons per frame. Bottom: pixel's dark current frequency spectrum, under the same conditions as above. Shown is the average of the Fourier analyses of 10 runs of 10,000 consecutive dark frames each. Equation (6) was fit to the spectrum, using $f_0 = 35.1$ Hz and dark current $I_{\text{ex}} = 50$ pA as found above. The fit yielded an internal gain G of 751 ± 6 (standard deviation).

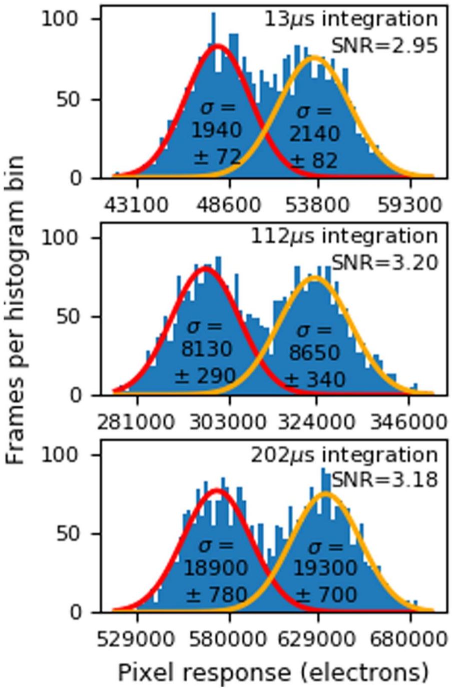

Fig. 3. The pixel in the previous figures was exposed to darkness and light, approximately 1500 frames each, at various integration times, and histograms were made of the output. Top: 13 μs. Center: 112 μs. Bottom: 202 μs. All measurements shared a frame rate of 4590 Hz, temperature of 240 K, and illumination of 1.1 pW spread uniformly across the pixel surface. The signal-to-noise ratio was nearly constant, despite the integration time varying by a factor of 15.

$$\mathrm{SNR}_{\mathrm{ROIC}} = \frac{I_s}{\sqrt{\frac{q}{T}F(I_S + I_D) + \left(\frac{q}{T}\frac{\mathrm{RN}}{G}\right)^2}}. \quad (8)$$

This follows the conventional wisdom that a longer integration time will yield higher SNR. However, in the detector-limited regime, $\mathrm{BW} = 1/(4\tau)$ instead and SNR becomes

$$\mathrm{SNR}_{\mathrm{det}} = \frac{I_s}{\sqrt{\frac{q}{2\tau}F(I_S + I_D) + \left(\frac{q}{T}\frac{\mathrm{RN}}{G}\right)^2}}. \quad (9)$$

Provided the read noise term is negligible compared to the shot-noise term, as occurs in the presence of high gain, the SNR in the detector-limited regime is constant regardless of the integration time used. We have observed this effect in our camera system, as shown in Fig. 3.

It is tempting to reduce integration time and increase frame rate to replace each frame with multiple shorter ones. The SNR of each would remain constant, so averaging multiple frames should increase sensitivity at no cost. However, this scheme erroneously assumes that consecutive frames are independent. In the detector-limited regime, frames occur within τ of each other and are thus correlated, so averaging produces no improvement. Similarly, while integration time can be reduced to small values, frame rate cannot be arbitrarily increased. The rate at which the camera system obtains information about a light signal will always be limited by the pixel response time. However, this constant-SNR effect can potentially increase the dynamic range of a detector-limited camera. By decreasing integration time, the number of signal electrons per frame reaching the ROIC integration capacitor can be made arbitrarily small without sacrificing sensitivity as long as the read noise term remains insignificant.

It is not enough to just make a better photodetector; the way it interacts with its readout systems must also be understood. We have shown that camera ROICs can introduce spectral filtering as a result of their charge integration process and that this changes the noise statistics of the system. In these cases, previous methods for calculated shot noise or internal gain will fail, and the methods we have presented must be used instead. As technology progresses from single-element photodetectors to full arrays and from low-gain to high-gain but slower photodetectors, we expect to encounter these situations increasingly often. This new understanding of Poisson statistics and the regimes in which they do and do not apply is critical to making the next generation of high-sensitivity cameras.

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