

Dynamic measurement and modeling of the Casimir force at the nanometer scale

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(Received 18 September 2009; accepted 9 January 2010; published online 10 February 2010)

We present a dynamic method for measurement of the Casimir force with an atomic force microscope (AFM) with a conventional AFM tip. With this method, originally based on the phase of vibration of the AFM tip, we are able to verify the Casimir force at distances of nearly 6 nm with an AFM tip that has a radius of curvature of nearly 100 nm. Until now dynamic methods have been done using large metal spheres at greater distances. Also presented is a theoretical model based on the harmonic oscillator, including nonidealities. This model accurately predicts the experimental data. © 2010 American Institute of Physics. [doi:10.1063/1.3302565]

The Casimir force is the force manifested between two bodies which are very close together and is caused by the quantum mechanical fluctuations of the vacuum in the space between those two bodies.¹ Hendrik Casimir proposed this force acting on two perfectly conducting parallel plates in 1948.² However, the bodies can be made of metal or semiconductor, the space between can be air, fluid, or vacuum, and the force can be either attractive or repulsive.³⁻⁵ Because of these possibilities this force has garnered recent attention due to the emergence of microelectromechanical systems (MEMS) and nanoelectromechanical systems (NEMS).^{6,7} The Casimir force has been thoroughly explored in a static fashion with setups involving extremely flat surfaces and large spheres.^{3-6,8,9} It has also been explored in a static fashion with normal atomic force microscope AFM tips.^{10,11} There has also been work done exploring the Casimir force in a dynamic mode but so far these methods have only employed the large spheres and at large distances.¹²⁻¹⁴ Here, we provide a method for measurement of the Casimir force with a conventional AFM tip (~100 nm radius) at a minimum distance of nearly 6 nm in noncontact mode. We believe that with the move from MEMS to NEMS, this will become a preferred modality for measuring the Casimir force because of the scale of the tip.

In order to understand and quantify our method, we have modeled the vibrating tip of an AFM with the equation of motion for a forced, damped harmonic oscillator with Casimir force (F_{Casimir}) and electrostatic force ($F_{\text{electrostatic}}$) terms added as follows:

$$m \frac{d^2x}{dt^2} + r \frac{dx}{dt} + kx = F_0 \cos(\omega_0 t) + F_{\text{electrostatic}} + F_{\text{Casimir}} \quad (1)$$

This is an equation with many parameters that must be obtained from experiment and $x(t)$ solved for numerically. We then compare the phase of $x(t)$ with the phase output given by the AFM controller. This phase is very sensitive to the forces on the right hand side of the equation including the Casimir force, even for tips of small radii. The parameters in this equation are the mass m , the damping factor r , the spring

constant k , and the magnitude of the driving force F_0 . Here, k may given by the manufacturer of the AFM tip. The mass and damping factor will be extracted by fitting an ideal curve to the frequency response of the disengaged tip as follows:

$$f(\omega) = \left[\omega \sqrt{r^2 + \left(\omega \times m - \frac{k}{\omega} \right)^2} \right]^{-1} \quad (2)$$

For the electrostatic force between the tip and the sample we use the formula for the electrostatic force between a sphere and a plate.⁹ $F_{\text{electrostatic}}$ is then given by the following:

$$F_{\text{electrostatic}} = 2\pi\epsilon_0 [V_{\text{dc}}^2 + \text{UnitStep}(t - t_0)(2V_{\text{dc}}V_{\text{ac}} + V_{\text{ac}}^2)] \sum_{n=1}^{\infty} \text{csch}(n\alpha) [\coth(\alpha) - n \times \coth(n\alpha)], \quad (3)$$

where $\alpha = \cosh^{-1}[1 + (d-x)/R]$, with R the radius of curvature of the sphere, ϵ_0 the permittivity of free space, d the distance between the plate and the sphere, V_{dc} the direct-current (dc) voltage between the tip and surface, and V_{ac} the alternating-current (ac) voltage between the tip and surface. We use the *UnitStep* function to simulate a square wave of an ac voltage. We choose a square wave because the sudden change in voltage provides the greatest initial change in phase. F_{Casimir} is given for a sphere above a plate by the following:¹¹

$$F_{\text{Casimir}} = \zeta(d) \frac{H_{\text{ts}} R}{6(d-x)^2}, \quad (4)$$

where H_{ts} is the Hamaker constant accounting for the dielectric properties of the tip and sample.¹⁰ H_{ts} can be calculated using various references for the various materials one chooses for the tip and sample. $\zeta(d)$ is a function that accounts for the surface roughness of the plate to fourth order as follows:³

$$\zeta(d) = 1 + 6 \left(\frac{A}{d-x} \right)^2 + 10 \left(\frac{A}{d-x} \right)^3 + 15 \left(\frac{A}{d-x} \right)^4, \quad (5)$$

where A is the surface roughness of the sample. To measure the surface roughness one can perform an AFM scan on an area of the sample and flatten it. Then the surface roughness

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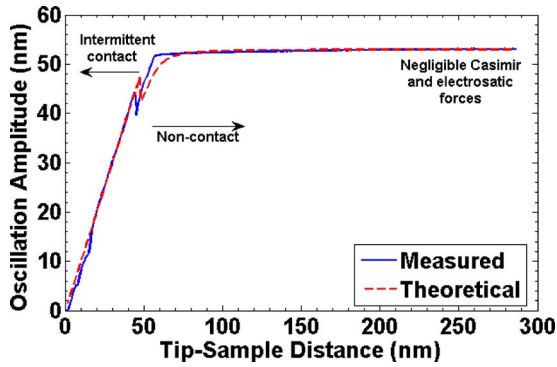


FIG. 1. (Color online) Tip oscillation amplitude vs tip-sample distance with a 0.5 V dc bias applied in both cases.

should be calculated on a region roughly the size of the diameter of the tip within the region of the AFM scan. The radius of curvature of the tip R can be measured with a scanning electron microscope (SEM). To find the distance of the tip to the sample d , one can perform an amplitude versus distance scan with the AFM while knowing the engaged amplitude of the tip, and find d . Finally, the amplitude of the driving force F_0 can be calculated using the amplitude of the solution to Eq. (1) and the measured amplitude of vibration at a far distance, where the Casimir and electrostatic forces are negligible.

We have performed the experiment using the method outlined above. Our experimental setup is quite similar to a Kelvin probe force microscope setup.¹⁵ At the core of the setup, we have our AFM in noncontact mode, with a conductive diamond tip and polished steel sample. Depending on the crystalline structure of the surfaces and the surface quality, the work function difference between the tip and sample can be positive, negative, or even zero, but is small for these two materials.^{16–18} Experimentally, we did not see an effect of this work function difference. There is a function generator connected to the tip that allows us to apply a changing bias between the tip and the sample, where the sample is grounded. An Agilent MAC-III AFM controller with lock-in amplifier provides amplitude and phase information when the AFM is in noncontact mode. The phase information is fed into an oscilloscope and a lock-in amplifier. The lock-in amplifier and oscilloscope are both synchronized with the function generator.

Some of the important parameters we have obtained experimentally are as follows: $F_0 \approx 0.988$ nano-Newtons, $d \approx 48.5$ nm, $R \approx 118$ nm, $k = 3$ N/m, and $\omega_0 = (100\,838 + 250) \times 2\pi$ rad/s. We have used $V_{dc} = 0.5$ V and $V_{ac} = 0.125$ V at a frequency of 100 Hz. Also, we have calculated H_{ts} to be to be around 8.16×10^{-19} J for existing references for steel^{11,19} and diamond.^{11,20,21} Finally, we have performed an AFM scan over a $4 \mu\text{m}^2$ area of the plate to calculate the surface roughness. We have found that the average surface roughness in a 234 nm^2 area of this area is 2.12 nm with a standard deviation of 1.34 nm.

We have found the amplitude of the tip oscillation at a constant 0.5 V bias, varying the distance from 1.5 to 286.5 nm and plotted the results in Fig. 1. It agrees well with the measured curve, also with a 0.5 V dc bias. The kink where it goes from noncontact to contact mode do not agree perfectly, however, because the position of this kink is variable by a few nanometers from one measurement to the next. When

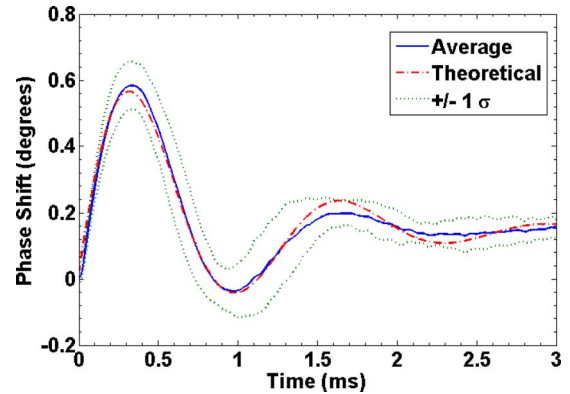


FIG. 2. (Color online) Phase change of the oscillation of tip as a function of time, both experimental and theoretical, with standard deviation shown.

the ac voltage of 0.125 V is turned on, we get the phase change seen in Fig. 2. This phase change agrees very well with that given by the model, which is quite impressive considering the phase of the vibrating AFM tip is very sensitive. In fact, the root mean square deviation of 1000 evenly spaced samples over the time period is only 0.019° . As you can see from the figure there is jitter in the secondary and tertiary peaks. Because an average of over 4000 measurements was taken for the composite figure and this jitter, the peaks in the figure are somewhat flattened compared to those given by the model. Because the model gives us the position of the tip as a function of time, we can calculate the minimum tip-sample separation. This minimum separation is calculated to be nearly 6.2 nm.

We have also varied the Hamaker constant H_{ts} and the surface roughness A in the model and calculated the root mean square deviation with the experiment and plotted it versus both parameters in Fig. 3. As you can see, the error is minimized in the $H_{ts} \approx 0.8 \times 10^{-18}$ J and $A \approx 2$ nm region. This error calculation was independent of our H_{ts} and surface roughness calculations, indicating that those calculations are close to their true values.

Finally, because our method is dynamic and not static like other methods, the model gives us a time-varying Casimir force. You can see this time-varying force plotted along with the time-varying driving force and electrostatic force

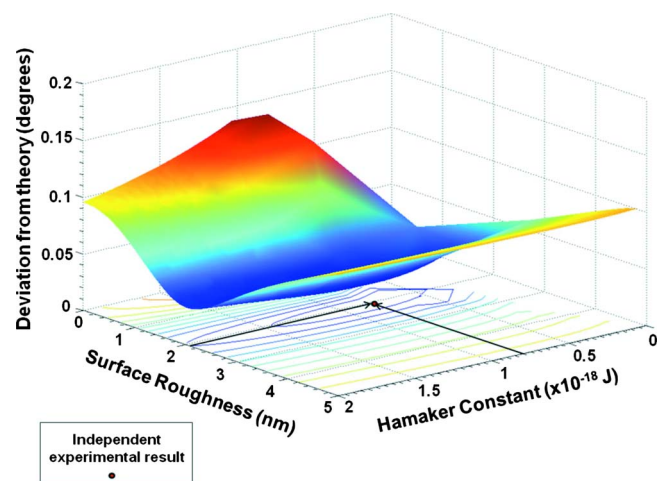


FIG. 3. (Color online) rms deviation of phase change of tip oscillation between theory and experiment as a function of surface roughness and Hamaker constant.

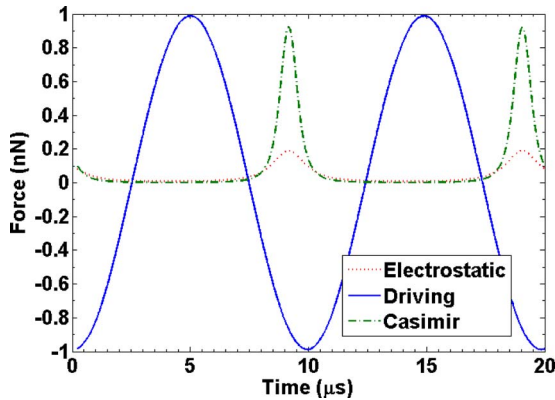


FIG. 4. (Color online) The three main forces acting on the tip as a function of time. The tip is being driven above the resonant frequency, and the voltage difference between tip and sample is 0.625 V.

plotted in Fig. 4. There are some interesting things to note from this figure, mostly, the magnitude and phase of the Casimir force. With a total applied voltage (ac+dc) of 0.625 V, the Casimir force has an over four times greater magnitude than the electrostatic force when the tip is at the minimum distance to the surface. Also, the magnitude of the Casimir force can be nearly as high as the magnitude used to drive the tip itself (about 1 nano-Newton). Possibly the most interesting thing about this figure is the phase of the Casimir and electrostatic forces with respect to the driving force. When the tip is driven above the resonant frequency, these forces are out of phase, meaning that when the Casimir and electrostatic forces are pulling in, the driving force is pulling the tip away from the surface. This is actually what keeps the tip in noncontact mode. Changing the driving frequency to be closer to the resonant frequency of the tip changes this phase, which therefore changes the point at which the tip goes from noncontact to contact mode.²²

We have presented a method for measurement of the Casimir force using an AFM in non-contact mode with an AFM tip of small radius. This method relies on the phase of vibration of the tip and a theoretical model based on the harmonic oscillator. This method deviates from earlier dy-

namic methods because it works at nanometer distances and with a conventional AFM tip. Lastly, we have verified independently the theoretically predicted effects of surface roughness and dielectric effects on the Casimir force. We believe that the presented method provides an efficient means of measuring the Casimir force with conventional AFM systems.

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